

Cambridge O Level

ADDITIONAL MATHEMATICS**4037/12**

Paper 1

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 3$	B1	
	$b = \frac{2}{3}$ oe	B1	
	$c = -4$	B1	

Question	Answer	Marks	Guidance
2	$rs = 6561$ isw	3	B2 for $\log_3 rs = 8$ or $\log_3 (rs)^2 = 16$ B1 for $2\log_9 s = \frac{2\log_3 s}{\log_3 9}$ soi
	Alternative 1		
	$rs = 6561$ isw	(3)	B2 for $\log_9 rs = 4$ or $\log_9 (rs)^2 = 8$ B1 for $\log_3 r = \frac{\log_9 r}{\log_9 3}$ soi
	Alternative 2		
	$rs = 6561$ isw	(3)	B2 for $\log rs = 8\log 3$ or $\log rs = 4\log 9$ for any base B1 for $\frac{\log r}{\log 3} + \frac{2\log s}{\log 9} = 8$ soi for any base
3	$\frac{dy}{dx} = \frac{1}{2} \sec^2 \frac{x}{2}$ or $\frac{1}{2 \cos^2 \left(\frac{x}{2}\right)}$	2	M1 for $k \sec^2 \frac{x}{2}$ or $\frac{k}{\cos^2 \left(\frac{x}{2}\right)}$, $k \neq \frac{1}{2}$
	$\sec^2 \frac{\pi}{6} = \frac{4}{3}$ or $\cos^2 \left(\frac{\pi}{6}\right) = \frac{3}{4}$ soi	B1	
	$\frac{dy}{dx} = \frac{2}{3}$	A1	
4(a)	6435	B1	
4(b)	9	B1	
4(c)	${}^6C_2 \times {}^5C_2$ (150)	B1	
	${}^6C_3 \times {}^5C_3 \times {}^4C_2$ (1200)	B1	
	${}^6C_4 \times {}^5C_4$ (75)	B1	
	Total: 1425	B1	Dep on all previous B marks

Question	Answer	Marks	Guidance
5(a)	$AD^2 = (\sqrt{7} - 2)^2 + (\sqrt{7} + 2)^2$ soi, oe	B1	
	$AD^2 = 11 - 4\sqrt{7} + 11 + 4\sqrt{7}$ or $AD = \sqrt{11 - 4\sqrt{7} + 11 + 4\sqrt{7}}$	B1	This is the minimum detail that is acceptable for this mark
	Perimeter = $18\sqrt{7} + \sqrt{22} - 14$ isw	B1	Allow if previous B mark not awarded because of lack of detail.
5(b)	$\frac{1}{2}(8\sqrt{7} - 7 + 9\sqrt{7} - 9)(\sqrt{7} + 2)$ oe	B1	
	$\frac{1}{2}(119 + 18\sqrt{7} - 32)$	B1	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw	B1	Allow if previous B mark not awarded because of lack of detail. Must be 2 separate terms.
	Alternative 1		
	Area of triangle = $\frac{3}{2}$	(B1)	
	Area of rectangle = $(8\sqrt{7} - 7)(\sqrt{7} + 2)$ $= 56 + 9\sqrt{7} - 14$	(B1)	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw	(B1)	Allow if previous B mark not awarded because of lack of detail.
	Alternative 2		
	Area of triangle = $\frac{3}{2}$	(B1)	
	Area of outer rectangle = $(9\sqrt{7} - 9)(\sqrt{7} + 2)$ $= 63 + 9\sqrt{7} - 18$	(B1)	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw		Allow if previous B mark not awarded because of lack of detail.

Question	Answer	Marks	Guidance
5(c)	$\cot \theta = \frac{\sqrt{7}+2}{9(\sqrt{7}-1)}$ or $\frac{1}{9(3-\sqrt{7})}$ oe	B1	May have factors of 9 multiplied out
	$\frac{(\sqrt{7}+2)}{9(\sqrt{7}-1)} \times \frac{(\sqrt{7}+1)}{(\sqrt{7}+1)}$ or $\frac{(3+\sqrt{7})}{9(3-\sqrt{7})(3+\sqrt{7})}$	M1	For rationalisation of <i>their</i> $\cot \theta$
	$\frac{\sqrt{7}}{18} + \frac{1}{6}$	A1	
	Alternative		
	$\cot \theta = \frac{\sqrt{7}+2}{9(\sqrt{7}-1)}$	(B1)	
	$\sqrt{7}+2 = (r\sqrt{7}+s)(9(\sqrt{7}-1))$ $1 = -9r+9s$ $2 = 63r-9s$	(M1)	Obtain 2 simultaneous equations from <i>their</i> $\cot \theta$. Allow one error
	$\frac{\sqrt{7}}{18} + \frac{1}{6}$	(A1)	
6(a)	$5\theta = 3.75$ so $\theta = 0.75$ AG	B1	Allow $\theta = \frac{3.75}{5}$ as a starting point

Question	Answer	Marks	Guidance
6(b)	$AB^2 = 5^2 + 5^2 - (2 \times 5 \times 5 \cos 1.75)$ $\sin(0.875) = \frac{0.5AB}{5}$ $\frac{AB}{\sin 1.75} = \frac{5}{\sin 0.696}$ or $DC^2 = 5^2 + 5^2 - (2 \times 5 \times 5 \cos 0.75)$ $\sin(0.375) = \frac{0.5DC}{5}$ $\frac{DC}{\sin 0.75} = \frac{5}{\sin 1.196}$	M1	For attempt to find the length of at least one chord. May be implied by a correct length.
	$AB = 7.67(5) \text{ soi}$	A1	
	$DC = 3.66(3) \text{ soi}$	A1	
	Arc length AD or $BC : 5 \times 0.5 \text{ soi oe}$	B1	Allow unsimplified
	16.3	A1	

Question	Answer	Marks	Guidance
Choose one scheme to mark to the advantage of the candidate			
6(c)	$\left(\frac{1}{2} \times 5^2 (1.75 - \sin 1.75)\right)$ $- \left(\frac{1}{2} \times 5^2 (0.75 - \sin 0.75)\right)$	M2	M2 for a fully correct method with no extra terms M1 for attempt at the area of at least one segment – may be implied by different methods
	8.7	A1	
	Alternative 1		
	Triangle ODC + sectors OCB and AOD only $\left(\frac{1}{2} \times 5^2 \times \sin 0.75\right) + 2 \left(\frac{1}{2} \times 5^2 \times 0.5\right)$	(M1)	
	Triangle OAB $\frac{1}{2} \times 5^2 \times \sin 1.75$	(M1)	
	8.7	(A1)	
	Alternative 2		
	Area of trapezium formed by Triangle ODC – triangle OPQ = $\left(\frac{1}{2} \times 5^2 \times \sin 0.75\right) - \left(\frac{1}{2} \times OP^2 \times \sin 0.75\right)$	(M1)	The points P and Q are where the lines OD and OC meet AB . Candidates may have different labels
	Area of APD and BQC = $2 \times \left(\left(\frac{1}{2} \times 5^2 \times 0.5\right) - \left(\frac{1}{2} \times 5 \times OP \times \sin 0.5\right) \right)$	(M1)	
	8.7	(A1)	
	Alternative 3		
	Area of trapezium $ABCD = \frac{1}{2}(AB + CD) \times 1.45$	(M1)	Allow 8.21
	area of 2 remaining shaded segments $2 \times \left(\left(\frac{1}{2} \times 5^2 \times 0.5\right) - \left(\frac{1}{2} \times 5^2 \times \sin 0.5\right) \right)$	(M1)	Allow M1 for one segment
	8.7	(A1)	

Question	Answer	Marks	Guidance
7(a)	$2x^2 - x(3x - 2) + (3x - 2)^2 = 2$ or $2\left(\frac{y+2}{3}\right)^2 - y\left(\frac{y+2}{3}\right) + y^2 = 2$	M1	For a quadratic equation in one variable
	$4x^2 - 5x + 1 = 0$ oe or $4y^2 + y - 5 = 0$ oe	A1	Allow multiples but must be a 3 term equation = 0
	$x = \frac{1}{4} \quad x = 1$ $y = -\frac{5}{4} \quad y = 1$	2	A1 for each correct pair
	Midpoint: $\left(\frac{5}{8}, -\frac{1}{8}\right)$	M1	Allow for use of <i>their</i> coordinates
	Gradient of $AB = 3$	M1	M1 for gradient of $AB = 3$ Allow for use of <i>their</i> coordinates
	Perpendicular gradient: $-\frac{1}{3}$	M1	Dep on previous M mark
	Perpendicular equation: $y + \frac{1}{8} = -\frac{1}{3}\left(x - \frac{5}{8}\right)$	M1	Allow for <i>their</i> perp gradient and <i>their</i> midpoint
	When $y = \frac{7}{8}$, $k = -\frac{19}{8}$ or -2.375 oe	A1	
7(b)	$\left(\frac{29}{8}, -\frac{9}{8}\right)$ or $(3.625, -1.125)$ oe	2	B1 for each coordinate
8(a)	$\frac{\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}(x+4) - (3x^2 - 5)^{\frac{1}{3}}}{(x+4)^2}$ or $\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}(x+4)^{-1}$ $+ (3x^2 - 5)^{\frac{1}{3}}(-(x+4)^{-2})$	3	B1 for $\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}$ soi M1 for attempt at differentiation of a quotient or valid product A1 for all other terms correct
	$(2x(x+4) - (3x^2 - 5))$	M1	M1 dep on previous M mark for simplification of <i>their</i> numerator Allow one sign slip but must be dealing with two quadratic terms.
	$\frac{-x^2 + 8x + 5}{(3x^2 - 5)^{\frac{2}{3}}(x+4)^2}$	A1	

Question	Answer	Marks	Guidance
8(b)	$-x^2 + 8x + 5 = 0$ oe, together with attempt to solve	M1	Allow for use of <i>their</i> quadratic numerator from part (a)
	$4 \pm \sqrt{21}$	2	A2 for both A1 for $\frac{8 + \sqrt{84}}{2}$ or $\frac{8 - \sqrt{84}}{2}$ oe
9(a)	Velocity vector = $\begin{pmatrix} -10 \\ 10.5 \end{pmatrix}$ or $\frac{1}{2} \begin{pmatrix} -20 \\ 21 \end{pmatrix}$	B2	B1 for $\left \begin{pmatrix} -20 \\ 21 \end{pmatrix} \right = 29$ soi or one correct element
9(b)	$\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} -10 \\ 10.5 \end{pmatrix} t$ oe	2	M1 FT on <i>their</i> velocity vector from part (a) if not correct.
9(c)	$\overrightarrow{PQ} = \begin{pmatrix} 5t - 4 \\ -3t - 2 \end{pmatrix}$ oe	2	Dep M1 must have M1 in part (b) for $\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t \right) - \text{their answer to part (b)}$ or <i>their</i> answer to part b $-\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t \right)$
	$PQ = \sqrt{(5t - 4)^2 + (-3t - 2)^2}$	M1	Dep on previous M mark for using <i>their</i> \overrightarrow{PQ}
	$PQ = \sqrt{34t^2 - 28t + 20}$	A1	
9(d)	For collision $34t^2 - 28t + 20 = 0$ soi	M1	Need to be using <i>their</i> quadratic PQ^2 from part (c) or forming a quadratic from <i>their</i> \overrightarrow{PQ} from part (c)
	Discriminant = -1936 So no real roots so no collision oe	A1	Discriminant must be correct, but allow unsimplified
10(a)(i)	$d = 2 \sin 2x$ soi	B1	
	$S_n = \frac{n}{2} (6 \sin 2x + 2(n-1) \sin 2x)$ or $S_n = \frac{n}{2} (3 \sin 2x + (2n+1) \sin 2x)$	M1	For correct use of the sum formula with <i>their</i> d
	$n(n+2) \sin 2x$	A1	

Question	Answer	Marks	Guidance
10(a)(ii)	$-220\sqrt{3}$ isw	2	Allow ‘starting again’ M1 for use of <i>their</i> answer to (a), but must be a correct method, $\sin\left(\frac{4\pi}{3}\right)$ needs to be evaluated to give an exact answer.
10(b)(i)	$2^{n-1} \ln 2y$ or $(\ln 2y)2^{n-1}$ or $\ln(2y)^{2^{n-1}}$	B2	B1 for $r = 2$
10(b)(ii)	$\ln 2y \frac{(1 - \text{their } r^n)}{(1 - \text{their } r)}$	M1	For correct use of sum formula with <i>their</i> non-logarithmic r
	$(2^n - 1) \ln 2y$ or $-1(1 - 2^n) \ln 2y$ or $\ln(2y)^{2^n - 1}$ or $-\ln 2y + 2^n \ln 2y$ oe isw when appropriate	A1	A0 if denominator is still present.
10(c)	$-\frac{3}{8} < w < \frac{5}{8}$ $-0.375 < w < 0.625$ $w < \frac{5}{8}$ and $w > -\frac{3}{8}$	3	B2 for $\left 2w - \frac{1}{4}\right < 1$ or $-1 < 2w - \frac{1}{4} < 1$ or $\frac{5}{8}$ and $-\frac{3}{8}$ seen B1 for $r = 2w - \frac{1}{4}$. May be seen in the sum to infinity formula
11(a)	$x + 2x \ln x$	2	M1 for attempt at product rule Allow unsimplified for A1
11(b)	$mx^2 \ln x$	M1	Mark final answer Dep on M1 in part (a)
	$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + c$ oe	2	A1 for $\frac{1}{2}x^2 \ln x$ oe A1 for $-\frac{x^2}{4} + c$ oe